# Intermediate Microeconomics

## Chapter 10: Intertemporal Choice

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时间就是金钱
效率就是生命
Time is
efficiency is life.


時間就是金錢
效率就是生命

# Intertemporal Choice

Persons often receive income in “lumps”, e.g., monthly salary

块

How is a lump of income spread over the following month (saving now for consumption later)?

Or how is consumption financed by borrowing now against income to be received at the end of the month?

# Present and Future Value

Begin with some simple financial arithmetic.

Take just two periods: 1 and 2.

Let r denote the interest rate per period.

# Future Value

终值 $m_t^T = m_t(1+r)^{T-t} = m_t\left(1+\frac{r}{N}\right)^{N(T-t)} \approx m_t e^{r(T-t)}$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{N}\right)^{N}.$$

Future value: If r=0.1 then $100 saved at the start of period 1 becomes $ 110 at the start of period 2.

The value next period of $1 saved not is the future value of that dollar.

# Present Value

现值

Suppose you can pay now to obtain \$1 at the start of next period.

What is the most you should pay?

\$1?

No, If you kept your \$1 now and saved it then at the start of next period you would have $\$(1+r)>\$1$, so paying \$1 now for \$1 next period is a bad deal.

# Intertemporal Budget Constraint

To start, let's ignore price effect by supposing that

$$\mathrm{p}_1 = \mathrm{p}_2 = \$1$$

The consumer's utility max problem:

$$\max \ \mathrm{u}(\mathrm{C}_1, \mathrm{C}_2)$$
$$s.t., \mathrm{p}_1\mathrm{C}_1 + \mathrm{p}_2\mathrm{C}_2 \leq \mathrm{m}_1 + \mathrm{m}_2$$

# Intertemporal Budget Constraint

To start, let's ignore price effect by supposing that

$$p_1 = p_2 = \$1$$

The consumer's utility max problem:

$$\max\ U(C_1, C_2)$$

~~$s.t., p_1C_1 + p_2C_2 \leq m_1 + m_2$~~

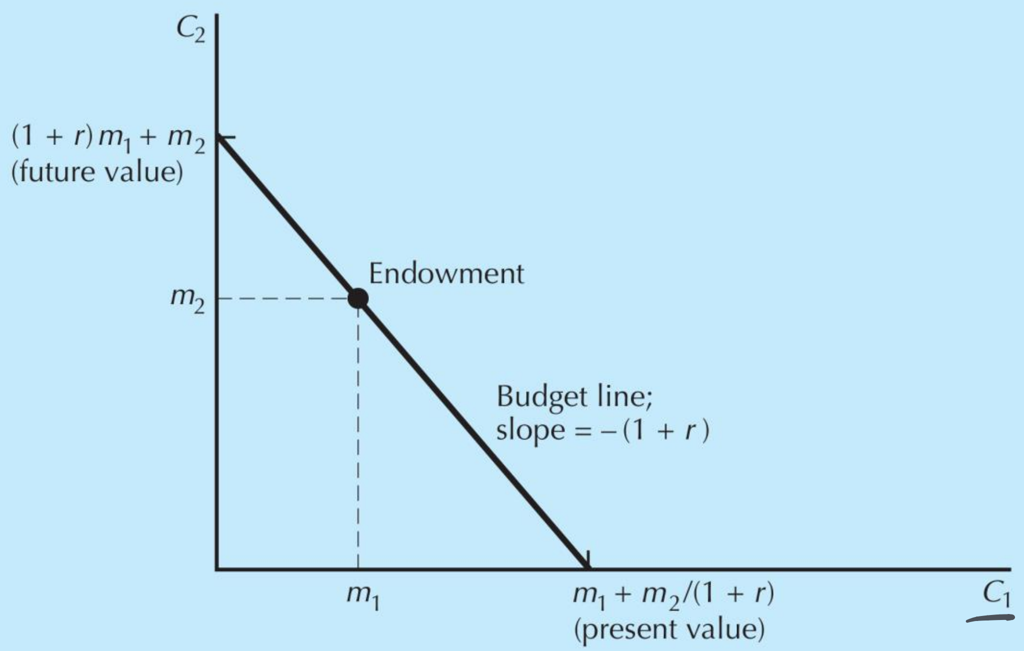
令 $p_1 = p_2 = 1$. $s.t.\ (1+r)C_1 + C_2 \leq (1+r)m_1 + m_2$

$$s.t., (1+r)p_1C_1 + p_2C_2 \leq (1+r)m_1 + m_2$$

or

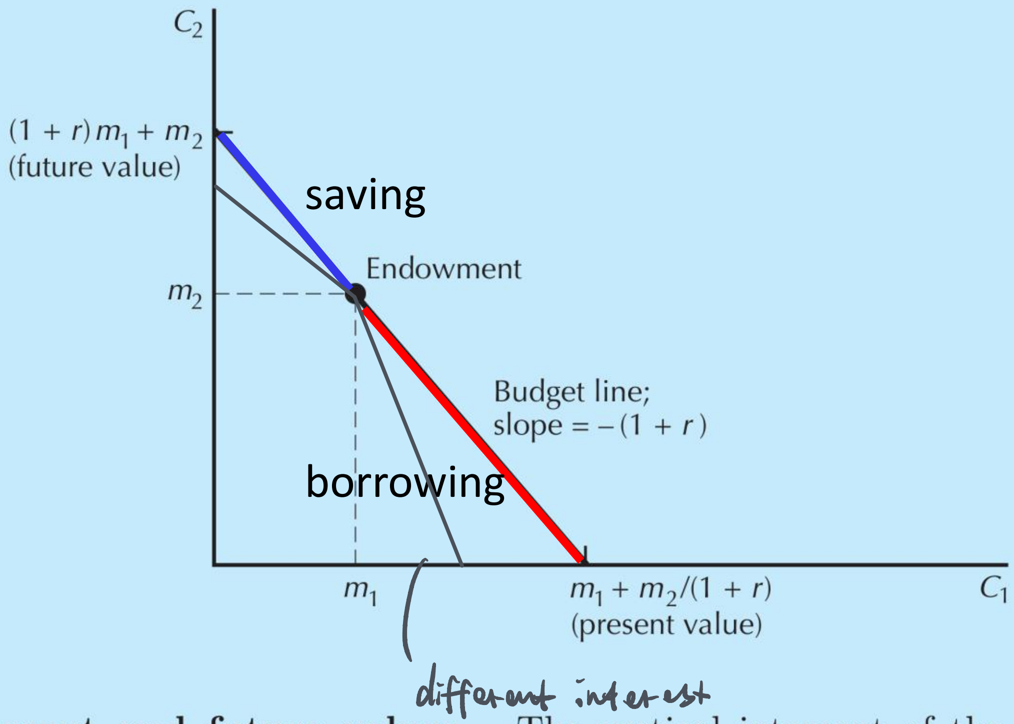
$$s.t., p_1C_1 + p_2/(1+r)C_2 \leq m_1 + m_2/(1+r)$$

# Intertemporal Budget Constraint




**Present and future values.** The vertical intercept of the budget line measures future value, and the horizontal intercept measures the present value.

# Intertemporal Budget Constraint




**Present and future values.** The vertical intercept of the budget line measures future value, and the horizontal intercept measures the present value.

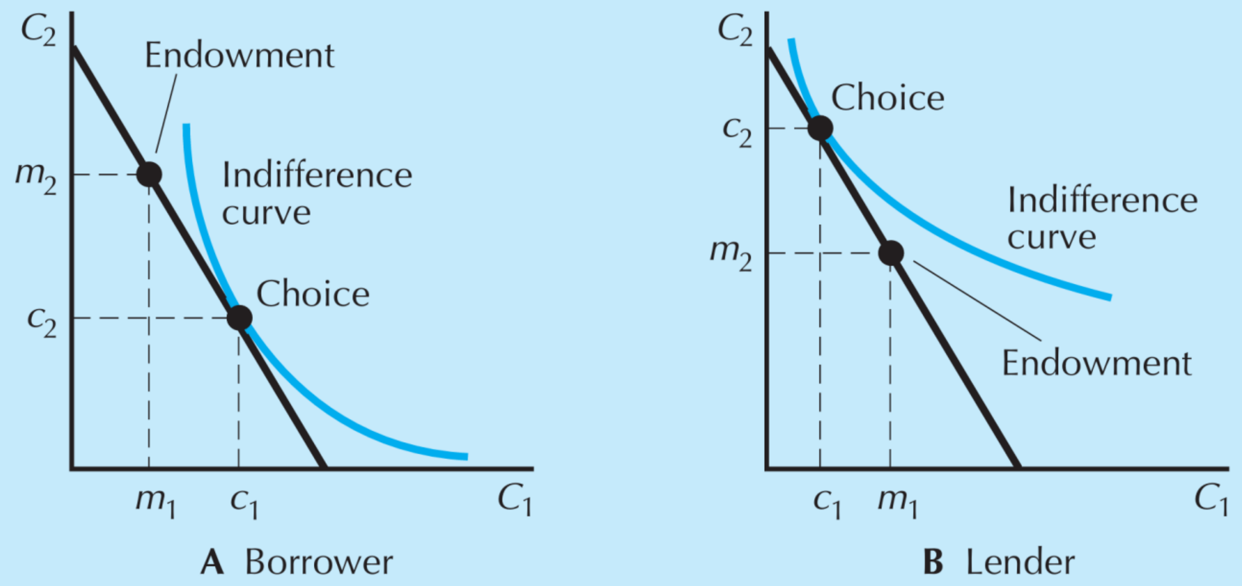
Consumer behavior:

1. Endowment
2. Saving area
3. Borrowing area

How does budget line change when deposit and loan interest rates differ?

# Intertemporal Choice

$u(C_1, C_2) = u(C_1) + \beta u(C_2)$  $\beta$: discount factor, $\beta < 1$ 贴现因子.




**Borrower and lender.** Panel A depicts a borrower, since $c_1 > m_1$, and panel B depicts a lender, since $c_1 < m_1$.

# Comparative Statics: Interest rate rises or falls

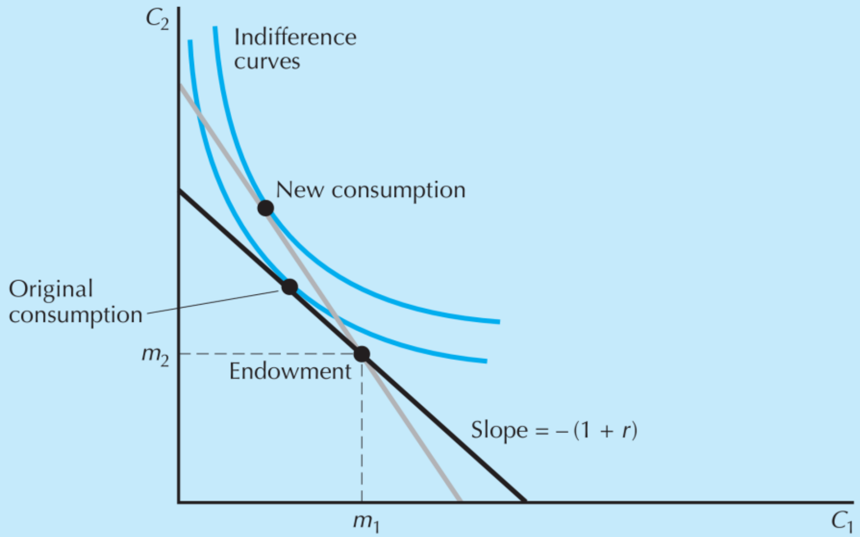
The slope of the budget constraint is

$$-(1+r)\frac{\mathrm{p}_1}{\mathrm{p}_2}$$

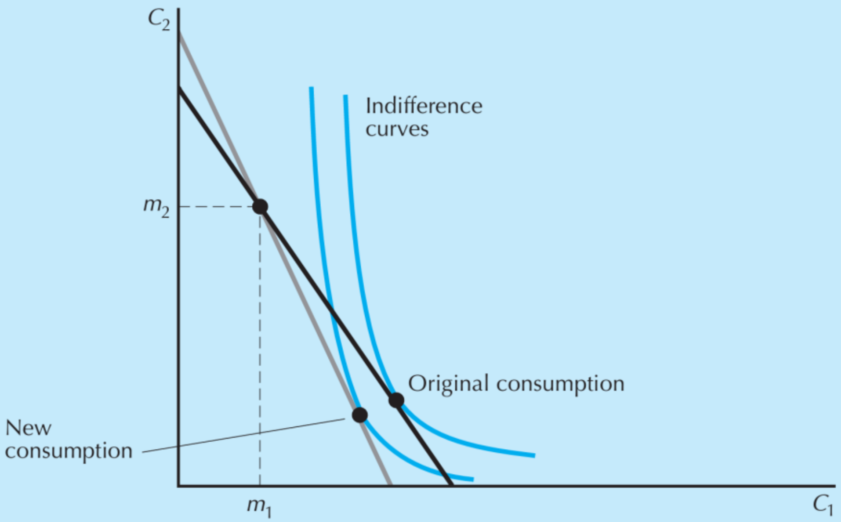
The constraint becomes flatter if the interest rate $r$ falls.

The constraint becomes steeper if the interest rate $r$ rises.

# Comparative Statics: Interest rate rises or falls




**If a person is a lender and the interest rate rises, he or she will remain a lender.** Increasing the interest rate pivots the budget line around the endowment to a steeper position; revealed preference implies that the new consumption bundle must lie to the left of the endowment.




**A borrower is made worse off by an increase in the interest rate.** When the interest rate facing a borrower increases and the consumer chooses to remain a borrower, he or she is certainly worse off.

# Effect on Consumption: Slutsky Equation

$$\frac{\partial \mathrm{C}_1^{\mathrm{t}}}{\partial r} = \frac{\partial \mathrm{C}_1^{\mathrm{s}}}{\partial r} + (\mathrm{m}_1 - \mathrm{C}_1)\frac{\partial \mathrm{C}_1^{\mathrm{m}}}{\partial r}$$

(?) (-) (?) (+)

If lender, $(\mathrm{m}_1 - \mathrm{C}_1) > 0$, total effect is negative

If borrower, $(\mathrm{m}_1 - \mathrm{C}_1) < 0$, total effect is ambiguous

# Comparative Statics: Price Inflation

Define the inflation rate by $\pi$ where

通货膨胀率

$$p_1(1+\pi) = p_2$$

For example,

$\pi = 0.2$ means 20% inflation, and

$\pi = 1.0$ means 100% inflation.

# Comparative Statics: Price Inflation

We lose nothing by setting $p_1 = 1$ so that $p_2 = 1 + \pi$

Then we can rewrite the budget constraint

$$p_1 c_1 + \frac{p_2}{1+r} c_2 = m_1 + \frac{m_2}{(1+r)}$$

as

$$\boxed{c_1 + \frac{1+\pi}{1+r} c_2 = m_1 + \frac{m_2}{(1+r)}}$$

rearranges to

$$c_2 = -\frac{1+r}{1+\pi} c_1 + \frac{1+r}{1+\pi} m_1 + \frac{1}{1+\pi} m_2$$

# Comparative Statics: Price Inflation

When there was no inflation ($\mathrm{p}_1 = \mathrm{p}_2 = 1$) the slope of the budget constraint was $-(1 + r)$

Now, with price inflation, the slope of the budget constraint is $-\frac{1+r}{1+\pi}$.
This can be rewritten as

$$-\frac{1+r}{1+\pi} = -(1+\rho)$$

$\rho$ is known as the real interest rate 实际利率 , $\rho = \frac{r-\pi}{1+\pi} \approx r - \pi$

$r$ is called as the nominal interest rate 名义利率

# Summary

1. The budget constraint for intertemporal consumption can be expressed in terms of present value or future value.

2. The comparative statics results derived earlier for general choice problems can be applied to intertemporal consumption as well.

3. The real rate of interest measures the extra consumption that you can get in the future by giving up some consumption today.

# Intermediate Microeconomics

## Chapter 12: Uncertainty

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# Uncertainty is Pervasive

What is uncertainty in economic systems?

—Tomorrow's prices

—Future wealth

—Future availability of commodities

—Present and future actions of other people

What are rational responses to uncertainty?

—Buying insurance (health, life, auto)

—A portfolio of contingent consumption goods

# States of Nature

Possible states of Nature:

—"car accident" (a)

—"no car accident" (na)

Accident occurs with probability $\pi_a$, does not with probability $\pi_{na}$ ;

$$\pi_a + \pi_{na} = 1$$

Accident causes a loss of $L

# State-Contingent Budget Constraint

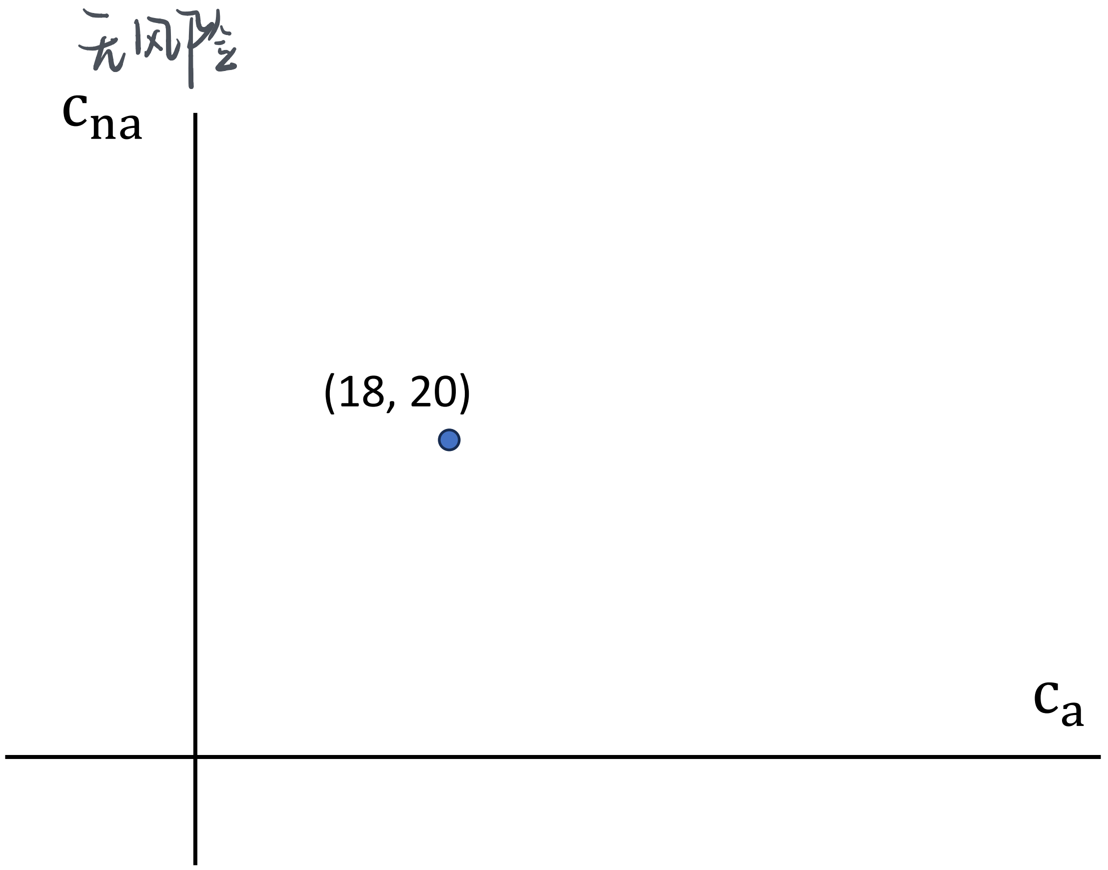
Each \$1 of accident insurance costs $\gamma$

Consumer has \$m of wealth

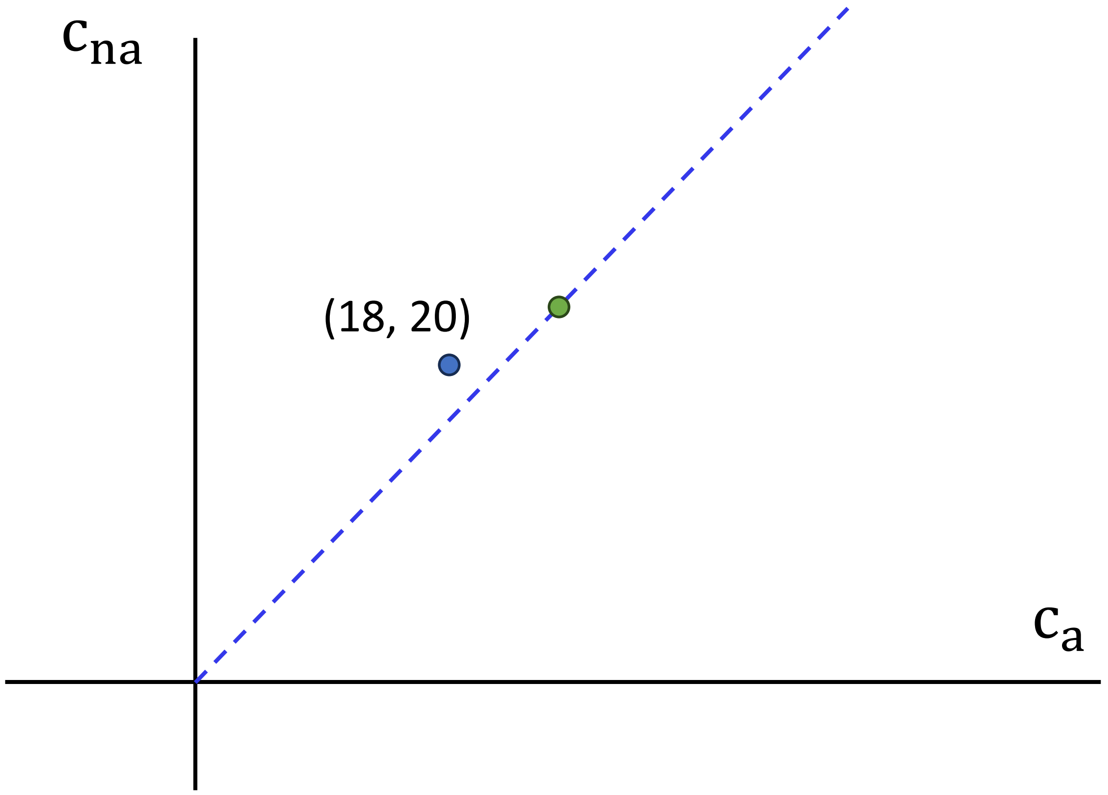
$c_{na}$ is consumption value in the no-accident state.

$c_a$ is consumption value in the accident state.

# State-Contingent Choice



# State-Contingent Choice



# State-Contingent Budget Constraint

What are the choices and expected outcomes under uncertain conditions?

Rely on the insurance market, or in other words, the risk trading market.

保费费率.

Trade potential losses at a premium rate of $\gamma$.

$c_{na} = M - \gamma K$

$c_a = M - L + (1-\gamma)K$

Assume insurance coverage $K \geq 0$,

$$c_{na} = M - \gamma K, c_a = M - \gamma K - L + K = M - L + (1-\gamma)K$$

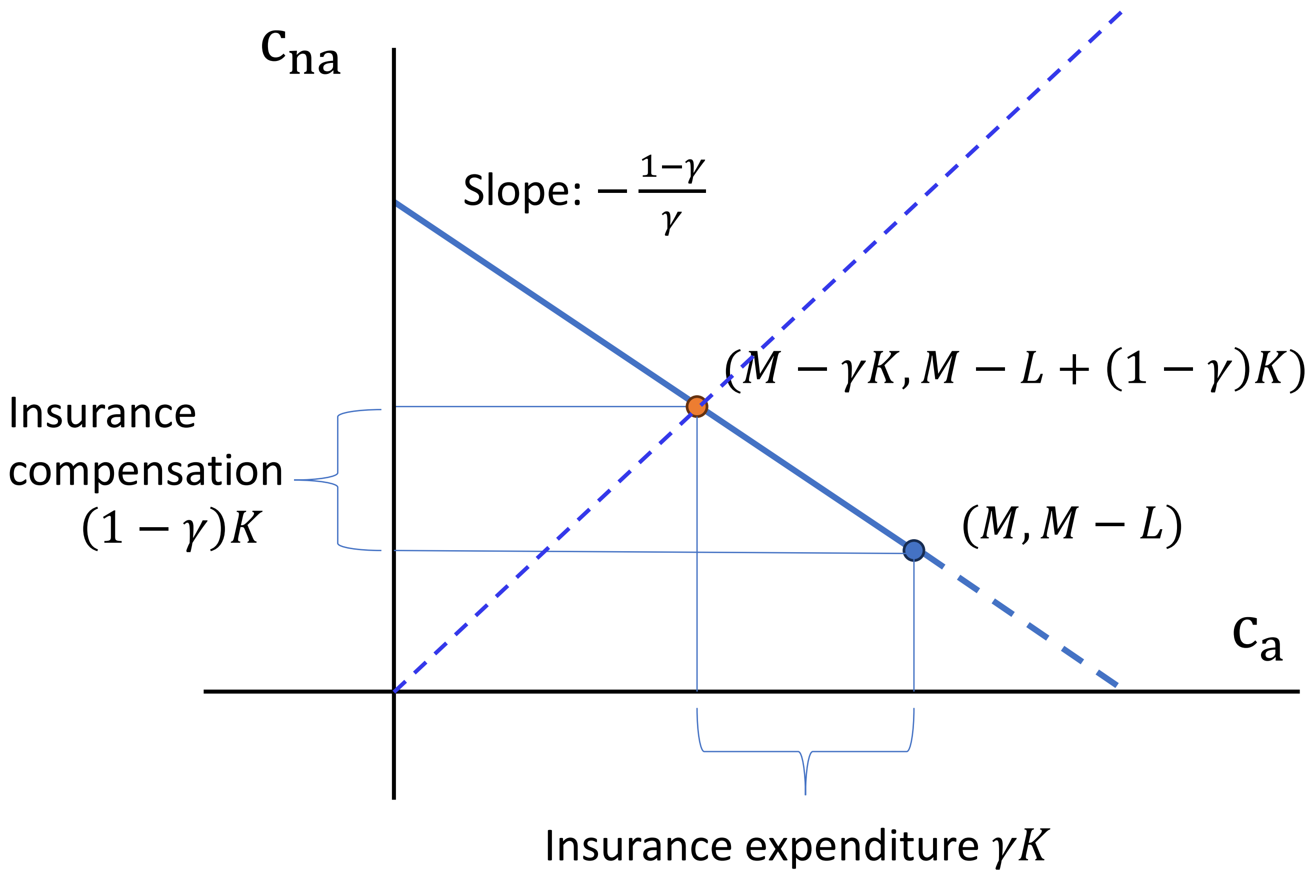
补偿.

Rearrange to

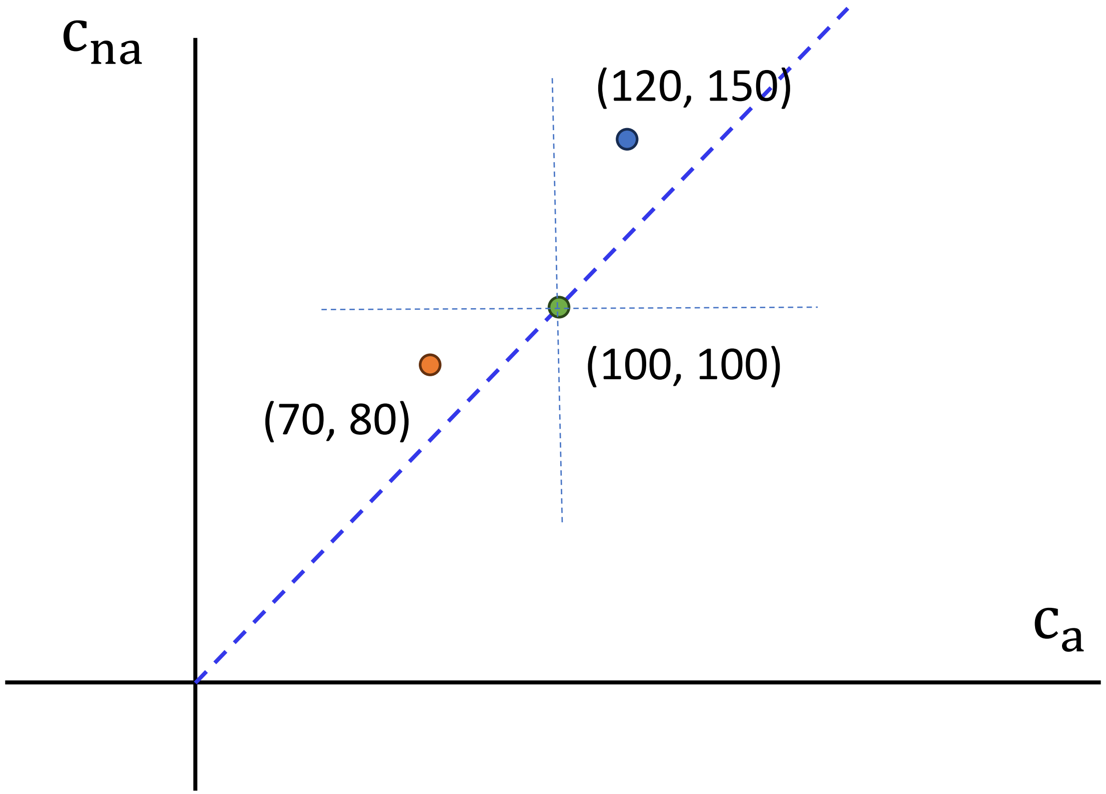
消去K:

$$(1-\gamma)c_{na} + \gamma c_a = M - \gamma L$$

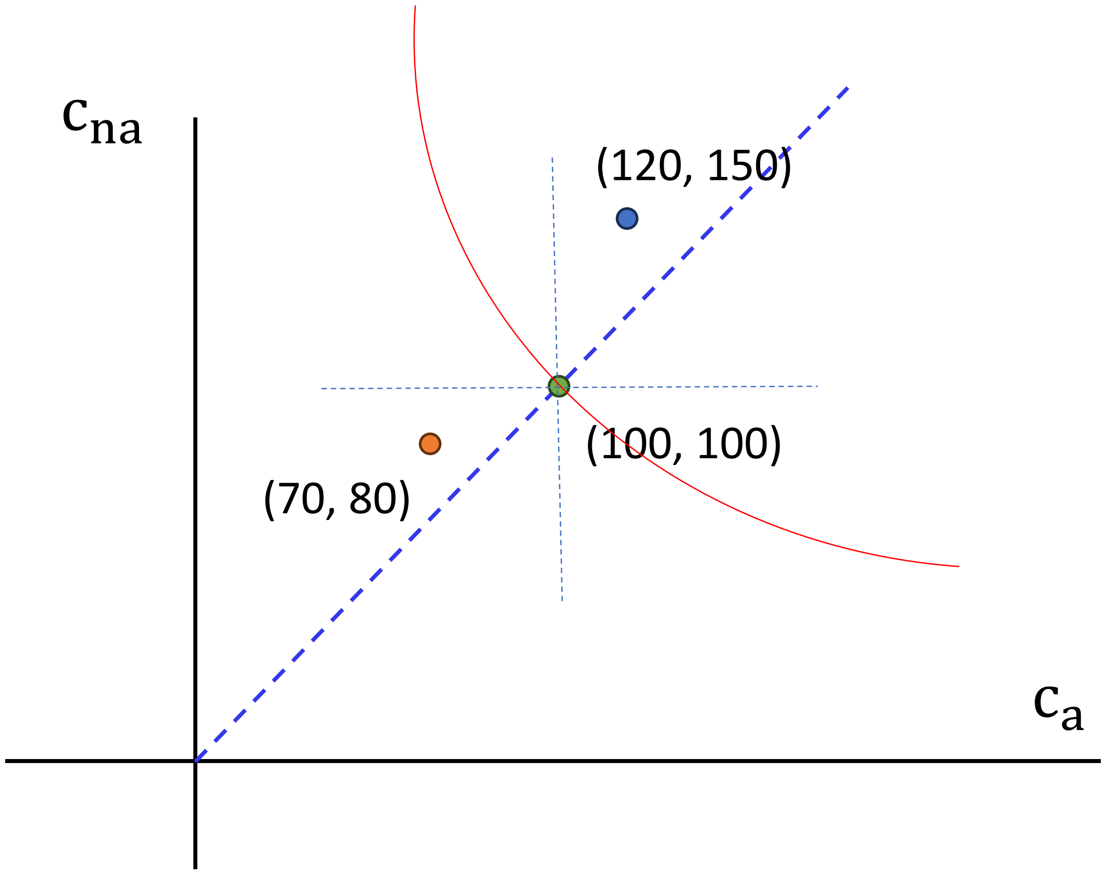
# State-Contingent Budget Constraint



# Preference under Uncertainty



# Utility Function




期望效用函数

Expected utility function:

Von-Neuman-Morgenstern utility function

$MRS = \frac{\partial u(c_a)/\partial c_a}{\partial u(c_{na})/\partial c_{na}} = \frac{\pi_a u'(c_a)}{\pi_{na} u'(c_{na})}$

$$\pi_a u(c_a) + \pi_{na} u(c_{na})$$

$$\frac{dc_{na}}{dc_a} = MRS = ?$$

$$MRS|_{c_a = c_a = c} = ?$$

# Risk Aversion 风险厌恶

An uncertain event: $(\pi_a c_a, \ \pi_{na} c_{na})$ 不确定

Risk averse: $u(\pi_a c_a + \pi_{na} c_{na}) > \pi_a \, u(c_a) + \pi_{na} u(c_{na})$

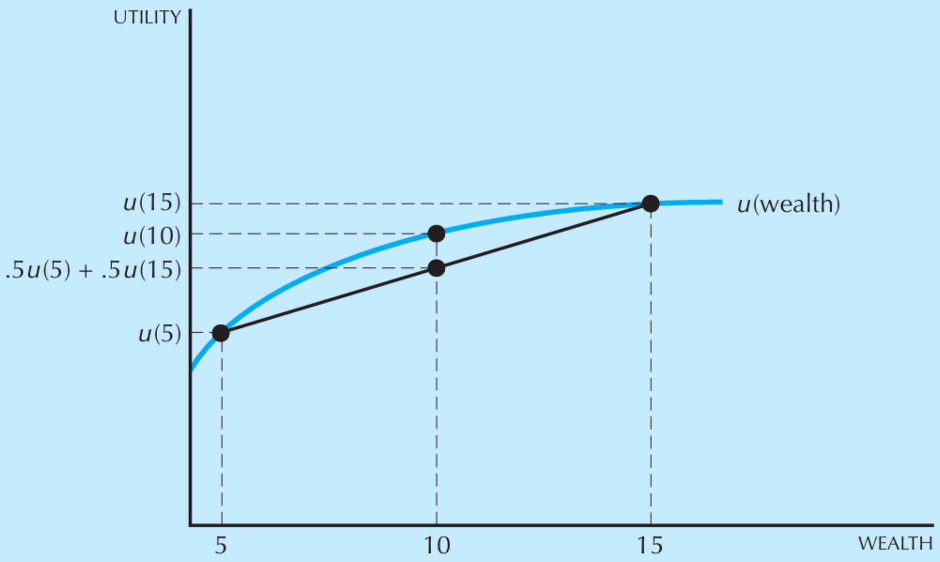
期望

随机

确定性

# Risk Aversion

convex




**Risk aversion.** For a risk-averse consumer the utility of the expected value of wealth, $u(10)$, is greater than the expected utility of wealth, $.5u(5) + .5u(15)$.

Suppose that a consumer currently has \$10 and is contemplating a gamble that gives him a 50% probability of winning \$5 and a 50% of losing \$5.

The expected value of his wealth is \$10, and the expected utility is

$$\frac{1}{2}u(\$15) + \frac{1}{2}u(\$5)$$

The utility of the expected value of wealth: $u\left(\frac{1}{2}15 + \frac{1}{2}5\right) = u(\$10)$

$$u(\$10) > \frac{1}{2}u(\$15) + \frac{1}{2}u(\$5)$$

# Other Risk Attitude

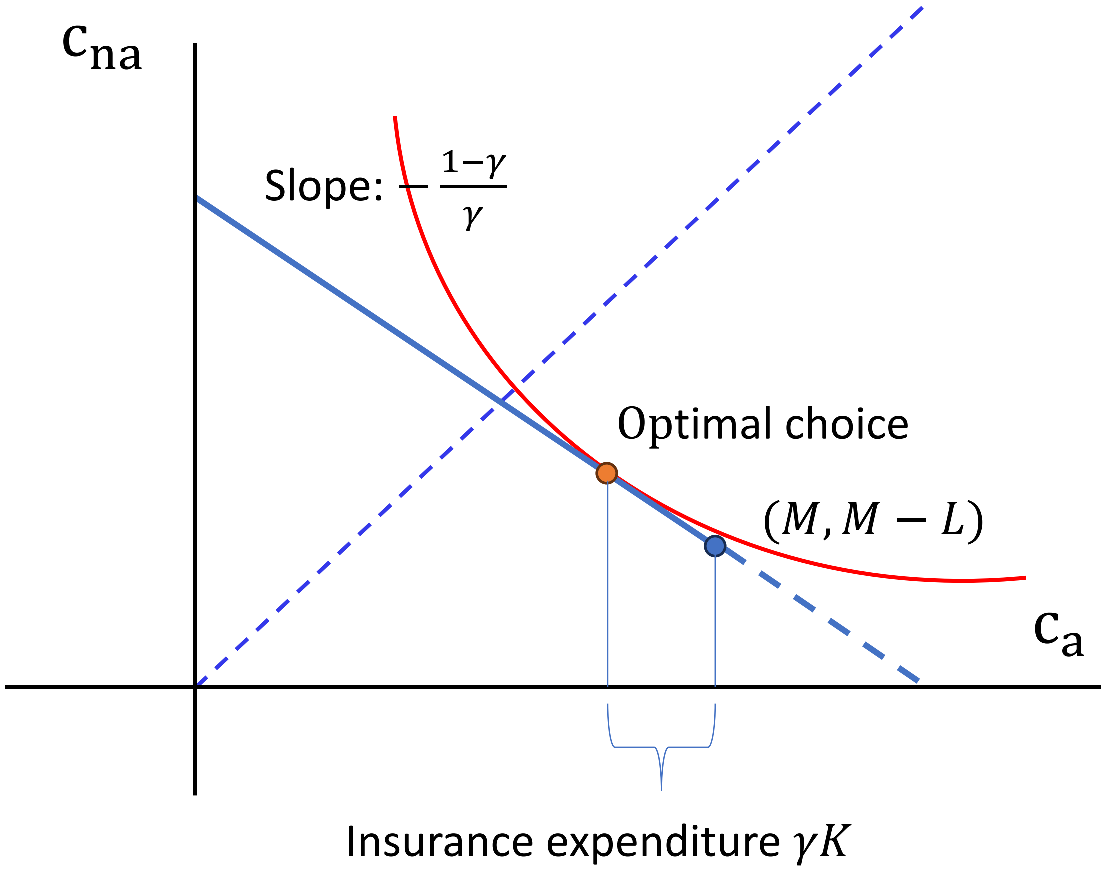
An uncertain event: $(\pi_a c_a,\ \pi_{na} c_{na})$

Risk averse: $u(\pi_a c_a + \pi_{na} c_{na}) > \pi_a\ u(c_a) + \pi_{na} u(c_{na})$

Risk lover: $u(\pi_a c_a + \pi_{na} c_{na}) < \pi_a\ u(c_a) + \pi_{na} u(c_{na})$

Risk neutral: $u(\pi_a c_a + \pi_{na} c_{na}) = \pi_a\ u(c_a) + \pi_{na} u(c_{na})$

# Optimal Choice



# An Example: The Demand for Insurance

Suppose the person had a wealth of \$35,000 and that he might incur a loss of \$10,000.

The probability of the loss was 1% ($\pi$), and it cost him $\gamma K$ to purchase $K$ dollars of insurance.

State 1: $c_1 = \$35000 - \gamma K$

State 2: $c_2 = \$35000 - \$10000 + K - \gamma K$

边际替代

$$MRS = -\frac{\pi u'(c_2)}{(1-\pi)u'(c_1)} = \frac{P_1}{P_2} = -\frac{-\gamma}{-(1-\gamma)}$$

Optimal choice: $\text{MRS} = -\frac{\pi u'(c_2)}{(1-\pi)u'(c_1)} = -\frac{\gamma}{1-\gamma}$

# An Example: The Demand for Insurance

Look at the insurance contract from the viewpoint of the insurance company. Its expected profit is

$$P = \gamma K - \pi K - (1-\pi) \times 0 = \gamma K - \pi K$$

Suppose that on the average the insurance company just breaks even on the contract. That is, they offer insurance at a "fair" rate, where

$$P = \gamma K - \pi K = 0$$

This implies $\gamma = \pi$

Then, MRS=$-\frac{\pi u'(c_2)}{(1-\pi)u'(c_1)} = -\frac{\gamma}{1-\gamma} = -\frac{\pi}{1-\pi} \Rightarrow u'(c_2) = u'(c_1)$

Therefore $c_1 = c_2 = \$35000 - \gamma K = \$35000 - \$10000 + K - \gamma K$,

$$K = \$10000$$

# Summary

1. Consumption in different states of nature can be viewed as consumption goods, and all the analysis of previous chapters can be applied to choice under uncertainty.
2. However, the utility function that summarizes choice behavior under uncertainty may have a special structure. In particular, if the utility function is linear in the probabilities, then the utility assigned to a gamble will just be the expected utility of the various outcomes.
3. The curvature of the expected utility function describes the consumer's attitudes toward risk. If it is concave, the consumer is a risk averter; and if it is convex, the consumer is a risk lover.